Math Virtual Learning

Precalculus with Trigonometry

May 20, 2020

INDEPENDENCE SCHOOL DISTRICT

Precalculus with Trigonometry
Lesson: May 20th, 2020

Objective/Learning Target:
Students will review how to use the Law of Sines and Law of Cosines to solve for angles and side lengths of non-right triangles.

Let's Get Started:

For a review of the Law of Sines, please watch the following videos.

Watch Video: Trigonometry Law of Sines/Sine Rule

Watch Video: Finding Angles Using the Sine Rule

Law of Sines

Used in AAS, ASA, and sometimes SSA situations.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

or

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Now watch the following video to review the Law of Cosines.

Watch Video: Law of Cosines

Law of Cosines

Used in SSS and SAS situations.

$$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

Example #1: AAS Situation

For the triangle in Figure 6.2, $C = 102°$, $B = 29°$, and $b = 28$ feet. Find the remaining angle and sides.

Solution

The third angle of the triangle is

$$\begin{aligned} A &= 180° - B - C \\ &= 180° - 29° - 102° \\ &= 49°. \end{aligned}$$

By the Law of Sines, you have

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

Using $b = 28$ produces

$$a = \frac{b}{\sin B}(\sin A) = \frac{28}{\sin 29°}(\sin 49°) \approx 43.59 \text{ feet}$$

and

$$c = \frac{b}{\sin B}(\sin C) = \frac{28}{\sin 29°}(\sin 102°) \approx 56.49 \text{ feet.}$$

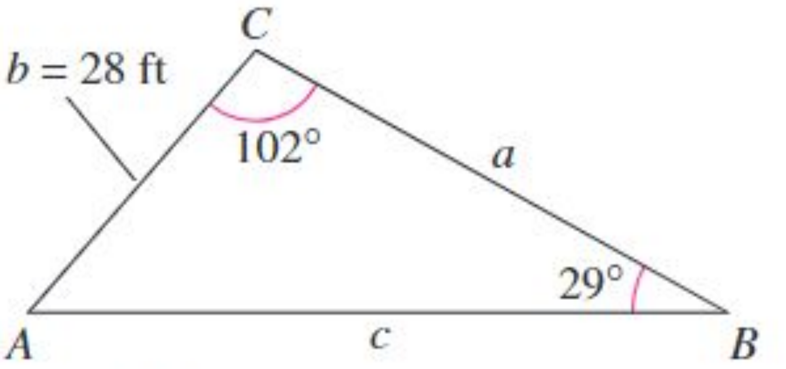


FIGURE 6.2

Example #2: ASA Situation

A pole tilts *toward* the sun at an 8° angle from the vertical, and it casts a 22-foot shadow. The angle of elevation from the tip of the shadow to the top of the pole is 43°. How tall is the pole?

Solution

From Figure 6.3, note that $A = 43°$ and $B = 90° + 8° = 98°$. So, the third angle is

$$C = 180° - A - B$$
$$= 180° - 43° - 98°$$
$$= 39°.$$

By the Law of Sines, you have

$$\frac{a}{\sin A} = \frac{c}{\sin C}.$$

Because $c = 22$ feet, the length of the pole is

$$a = \frac{c}{\sin C}(\sin A) = \frac{22}{\sin 39°}(\sin 43°) \approx 23.84 \text{ feet.}$$

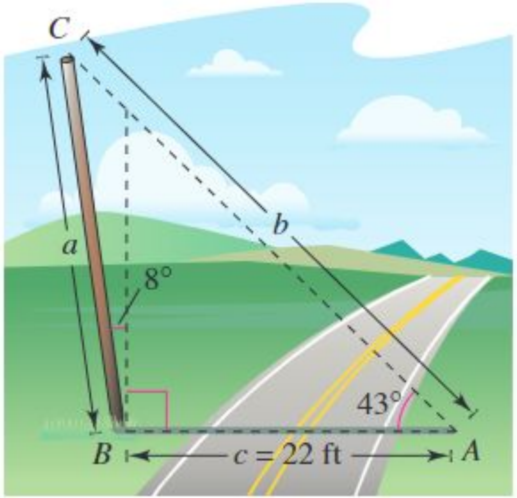
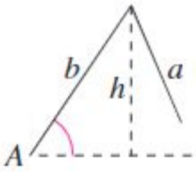
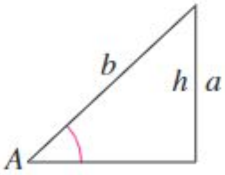
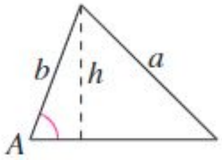
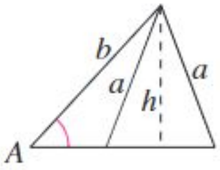
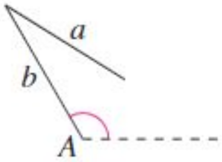
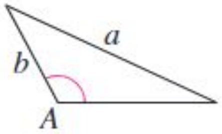


FIGURE 6.3

The Ambiguous Case (SSA)

Consider a triangle in which you are given a, b, and A. $(h = b \sin A)$

	A is acute.	A is acute.	A is acute.	A is acute.	A is obtuse.	A is obtuse.
Sketch						
Necessary condition	$a < h$	$a = h$	$a \geq b$	$h < a < b$	$a \leq b$	$a > b$
Triangles possible	None	One	One	Two	None	One

Example #3: SSA Situation

Find two triangles for which $a = 12$ meters, $b = 31$ meters, and $A = 20.5°$.

Solution

By the Law of Sines, you have

$$\frac{\sin B}{b} = \frac{\sin A}{a} \qquad \text{Reciprocal form}$$

$$\sin B = b\left(\frac{\sin A}{a}\right) = 31\left(\frac{\sin 20.5°}{12}\right) \approx 0.9047.$$

There are two angles, $B_1 \approx 64.8°$ and $B_2 \approx 180° - 64.8° = 115.2°$, between $0°$ and $180°$ whose sine is 0.9047. For $B_1 \approx 64.8°$, you obtain

$$C \approx 180° - 20.5° - 64.8° = 94.7°$$

$$c = \frac{a}{\sin A}(\sin C) = \frac{12}{\sin 20.5°}(\sin 94.7°) \approx 34.15 \text{ meters.}$$

For $B_2 \approx 115.2°$, you obtain

$$C \approx 180° - 20.5° - 115.2° = 44.3°$$

$$c = \frac{a}{\sin A}(\sin C) = \frac{12}{\sin 20.5°}(\sin 44.3°) \approx 23.93 \text{ meters.}$$

The resulting triangles are shown in Figure 6.6.

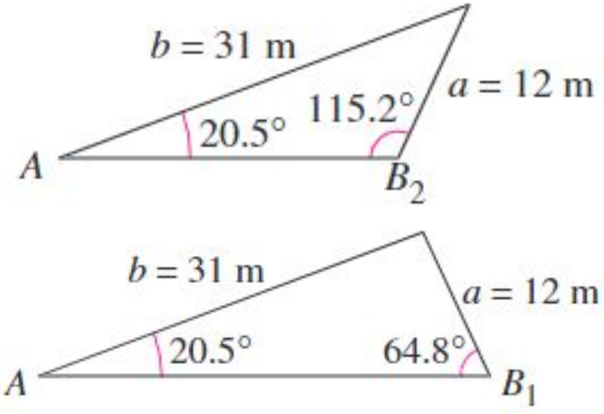


FIGURE 6.6

Example #4: SSS Situation

Find the three angles of the triangle in Figure 6.11.

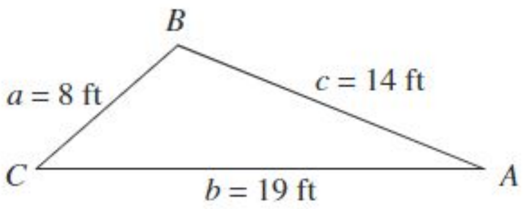


FIGURE 6.11

Solution

It is a good idea first to find the angle opposite the longest side—side b in this case. Using the alternative form of the Law of Cosines, you find that

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{8^2 + 14^2 - 19^2}{2(8)(14)} \approx -0.45089.$$

Because $\cos B$ is negative, you know that B is an *obtuse* angle given by $B \approx 116.80°$. At this point, it is simpler to use the Law of Sines to determine A.

$$\sin A = a\left(\frac{\sin B}{b}\right) \approx 8\left(\frac{\sin 116.80°}{19}\right) \approx 0.37583$$

You know that A must be acute because B is obtuse, and a triangle can have, at most, one obtuse angle. So, $A \approx 22.08°$ and $C \approx 180° - 22.08° - 116.80° = 41.12°$.

Example #5: **SAS Situation**

Find the remaining angles and side of the triangle in Figure 6.12.

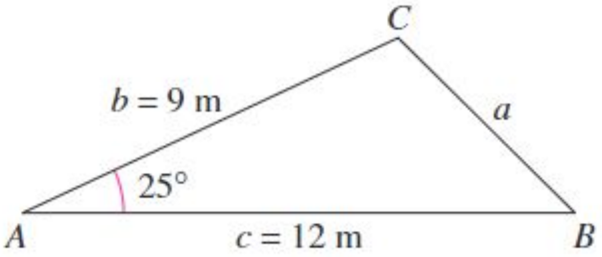


FIGURE 6.12

Solution

Use the Law of Cosines to find the unknown side a in the figure.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 9^2 + 12^2 - 2(9)(12) \cos 25°$$

$$a^2 \approx 29.2375$$

$$a \approx 5.4072$$

Solution Continues on Next Page

Example #5: Continued

Because $a \approx 5.4072$ meters, you now know the ratio $(\sin A)/a$ and you can use the reciprocal form of the Law of Sines to solve for B.

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\sin B = b\left(\frac{\sin A}{a}\right)$$

$$= 9\left(\frac{\sin 25}{5.4072}\right)$$

$$\approx 0.7034$$

There are two angles between $0°$ and $180°$ whose sine is 0.7034, $B_1 \approx 44.7°$ and $B_2 \approx 180° - 44.7° = 135.3°$.

For $B_1 \approx 44.7°$,

$$C_1 \approx 180° - 25° - 44.7° = 110.3°.$$

For $B_2 \approx 135.3°$,

$$C_2 \approx 180° - 25° - 135.3° = 19.7°.$$

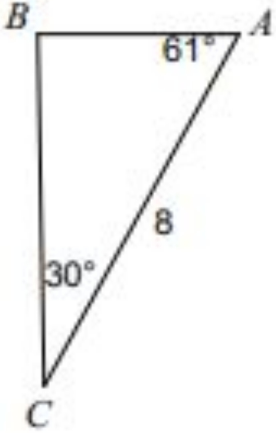
Because side c is the longest side of the triangle, C must be the largest angle of the triangle. So, $B \approx 44.7°$ and $C \approx 110.3°$.

Practice

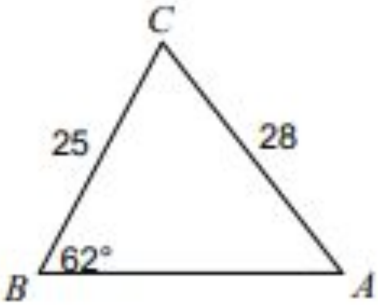
On a separate piece of paper, do each of the following problems. Answers will be provided on the next page.

Find each measurement indicated. Round your answers to the nearest tenth.

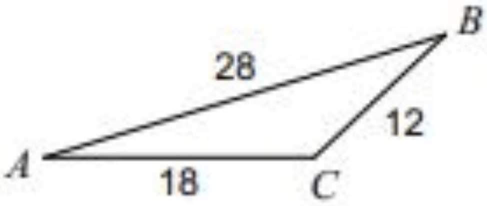
1) Find BC



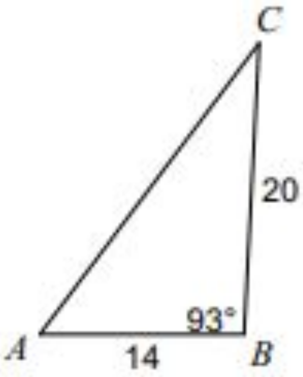
2) Find $m\angle A$



3) Find $m\angle C$



4) Find AC

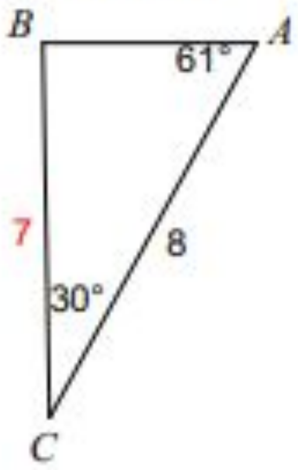


Practice - **ANSWERS**

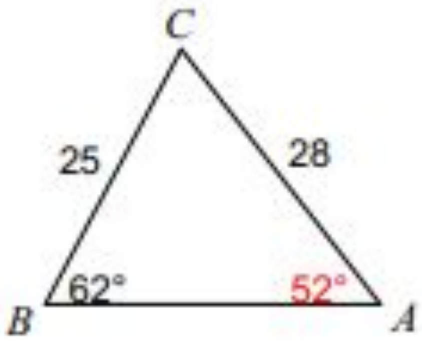
On a separate piece of paper, do each of the following problems. Answers will be provided on the next page.

Find each measurement indicated. Round your answers to the nearest tenth.

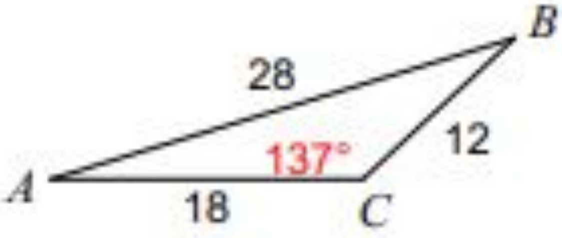
1) Find BC



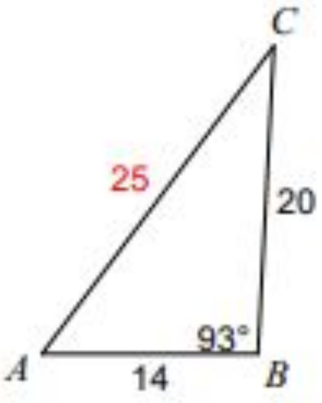
2) Find $m\angle A$



3) Find $m\angle C$



4) Find AC



Additional Resource Videos:

[Law of Sines and Cosines, explanation]

[Law of Sines and Law of Cosines]

Additional Practice:

[Law of Sines & Cosines Review]

[Sine Law & Cosine Law - Extra Practice]